Mathematics and Religion: The Intersection is Not Empty

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Mathematics and Religion

A Discussion in Three Parts: Part I Numbers in Scripture Part II **Mathematicians** Part III **Religious Use of Mathematics**

Mathematics and Religion

Pop Quiz

- 1. What is the biblical value of pi?
- 2. How many fish were caught in an unbroken net?
- 3. How many goats did Jacob give to Esau? How many goats should Esau have sent to Jacob in order to confirm their friendship?
- 4. In what context does the number 666 appear in the Bible?
- 5. What is the largest number mentioned in the Bible?
- 6. What is the smallest number not mentioned in the Bible?
- 7. Identify the number 969. What are some of its mathematical characteristics?

1. What is the biblical value of π ?

A structure in King Solomon's temple described:

"And he made the molten sea of **ten cubits from brim to brim**, round in compass, and the height thereof was five cubits; and a <u>line of thirty cubits did compass it</u> **round** about." I Kings 7: 23, II Chronicles 4:2

10 Cubits 30 Cubits Circumference π Diameter $=\frac{30}{10}=3$ π

Visually:

Elijah of Vilna "Gaon of Vilna" Late 18th century Polish rabbi

Hebrew word for "line measure" was written differently in each of the verses.

With gematria, demonstrated that the biblical estimation is correct to four decimal places.



Gematria is the calculation of the numerical equivalence of letters, words, or phrases, and, on that basis, gaining, insight into interrelation of different concepts and exploring the interrelationship between words and ideas.

I Kings 7:23

717 5 + 6 + 100 = 111

II Chronicles 4:2

T 6 + 100 = 106

Creating a ratio:

$$\frac{111}{106} = 1.0472$$

3*1.0472 = 3.1416

2. How many fish were caught in an unbroken net?

153

"Simon Peter went up and drew the net to land, full of large fish, a hundred and fiftythree, and although there were so many, the net was not torn."

John 21:11

The number 153 has many interesting properties

153 is a triangular number. Triangular numbers are the sum of the integers from 1 to n. Examples of Triangular numbers

1: *	3:	6: *	10:*	15: *	*		
	*	* *	* *	*	*	*	
	* *		* * *	*	*	* *	
		* * *	* * *	* *	*	* *	*
1	1+2	1+2+3	1+2+3	+4 1-	⊦2+	·3+4+	-5

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Numbers in Scripture 153 = 1 + 2 + 3 + + 16 + 17

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a triangle with 17 rows.

 $153 = 1^3 + 5^3 + 3^3$, that is 153 can be written as the sum of the cubes of its digits.

Question to Ponder:

If you think that this is a common feature of numbers, then see how many other three digit numbers you can find with this property.

There are four three digit numbers that can be expressed as the cubes of their digits.

$$3^{3} + 7^{3} + 0^{3} = 370$$

 $3^{3} + 7^{3} + 1^{3} = 371$
 $4^{3} + 0^{3} + 7^{3} = 407$
 $1^{3} + 5^{3} + 3^{3} = 153$

 $153 = 12^2 + 3^2$, that is 153 can be written as the sum of two squares.

Not all numbers have this property.

The numbers 3 and 12 themselves have special significance.

- Factorial!
- $5! = 5 \times 4 \times 3 \times 2 \times 1$

153 = 1! + 2! + 3! + 4! + 5!= 1 + 2 + 6 + 24 + 120

Binary Numbers (Base 2 system)

153 in binary is 10011001.

This number is a palindrome, i.e., it reads the same forward and backward.

The Greek word for fishes "ichthus" IX Θ Y Σ , by Gematria is 1224 or 8 x 153.

Numbers in Scripture The expression, בניהאלהים "Sons of God" has gematria of 153

ם יה אל היה ים 1 כ 40 + 10 + 5 + 30 + 1 + 5 + 10 + 50 + 2= 153

- 3. How many goats did Jacob give to Esau? How many goats should Esau have sent to Jacob in order to confirm their friendship?
- " …… Then he selected from what he had with him a present for his brother Esau: two hundred female goats and twenty male goats…." Genesis 32:13

Amicable Numbers

Amicable numbers: The pair of numbers *a* and *b* are called **Amicable** if the proper divisors of *a* sum to *b* and if the proper divisors of *b* sum to *a*.

Number	rs in S	Scripture	ļ
Proper	<u>divisc</u>	ors of 220	
	1		
	2	110	
	4	55	
	5	44	
	10	22	
	<u>11</u>	20	
Column Totals	33	+ 251	= <u>284</u>

Numbers in Scripture Proper divisors of 284 1 142 2 4 71

1 + 2 + 4 + 71 + 142 = 220

220 and 284 are amicable numbers.

Therefore 220 and 284 are a pair of amicable numbers and hence Esau should send Jacob 284 animals as a sign of friendship.

(17296, 18416) Fermat 1636

(9363584, 9437056) Descartes 1638

(1184, 1210) Nicolo Paganini, age 16, 1866

(42262694537514864075544955198125, 42405817271188606697466971841875) Battiato and Borho (1988)

4. In what context does the number 666 appear in the Bible?

"Here is wisdom, let him who has understanding calculate the number of the beast, for the number is that of a man, and the number is six hundred and sixty six." Revelation 13:28

666 is a palindrome

666 = 1 + 2 + 3 + 4 + ... + 33 + 34 + 35 + 36 Therefore it is triangular.

In the Roman numeration system, the first six numbers are I, V, X, L, C, D D 500

- C 100
- L 50
- X 10
- V 5 <u>I 1</u>

666

Magic Squares contain rows and columns of whole numbers with the property that every row, and every column, and the diagonals sum to the same value.

666 defines one such magic square.

To begin creating a magic square we must first determine the magic sum. For an $n \times n$ square, sum the numbers from 1 to m, where $m = n^2$. is the total number of entries in the square. Then divide by n.

Ex. 3x3 has 9 numbers.

1+2+3+4+5+6+7+8+9 = 45.

45/3 = 15.

Each row, column and diagonal must sum to 15.

- To find the magic sum for a 6x6 magic square, begin by summing the integers 1 to 36, which equals 666. 666/6 = 111Sum is 111.
- 11 8 27 28 7 30 19 14 16 15 23 24 22 21 17 18 20 13 10 9 25 29 26 12

33

6 32 3 34

36 5 33 4 2 31

However the number 666 has its most intriguing application in the sport of "beasting" (i.e. showing that someone has the number of the beast). This was great fun during the Reformation.Example 1: Beasting Pope Leo X

Example 2: Beasting Martin Luther

A-IK-ST-Z1-910-90100-500

Latin lacks J, W, V≡U, thus MARTIN LVTERA 30+1+80+100+9+40 + 20+200+100+5+80+1 = 666

5. What is the largest number mentioned in the Bible?

Myriad is Greek for 10,000

- "... thousand thousands ministered unto him, and ten thousand times ten thousand stood before him..." Daniel 7:10
- "And the number of the army of the horsemen were two hundred thousand thousand... Rev 9:16

Two myriad myriad (2 x 10000 x 10000)is the largest number in the bible.

This is also the largest number named by the Ancient Greeks.
6. What is the smallest number not mentioned in the bible?

43

7. Identify the number 969. What are some of its mathematical characteristics?

" So all the days of Methuselah were nine hundred and sixty-nine years." Genesis 5:27

969 is a palindrome

969 is the sum of the first 17 triangular numbers.

1+3+6+10+15+21+28+36+45+55+66+78+91+105+120+136+153 = 969

Summing triangular numbers creates tetrahedral numbers



969 is a tetrahedral number.

Recall the triangular numbers



The sum of two consecutive triangular numbers is a square.



Regrouping

- 1 + (3+6) + (10+15) + (21+28) + (36+45) + (55+66) + (78+91) + (105+120) + (136+153)
- 1 + 9 + 25 + 49 + 81 + 121 + 169 + 225 + 289
- $1^2 + 3^2 + 5^2 + 7^2 + 9^2 + 11^2 + 13^2 + 15^2 + 17^2$

969 is the sum of the square of the odd integers from 1 to 17.

969 can be expressed four ways as the difference of two squares.

$$969 = 35^{2} - 16^{2}$$
$$= 37^{2} - 20^{2}$$
$$= 163^{2} - 160^{2}$$
$$= 485^{2} - 484^{2}$$

Flipping the digits gives 696

696 is also a palindrome

696 can be expressed four ways as the difference of two squares.

$$696 = 175^{2} - 173^{2}$$
$$= 89^{2} - 85^{2}$$
$$= 61^{2} - 55^{2}$$
$$= 35^{2} - 23^{2}$$

H.S.M. Coxeter observed that in addition to the above the bible states that at age 187 Methuselah begat Lamech and that when Lamech was 182 years old he begat Noah and Noah was 600 years old when the flood began. Using this to calculate Methuselah's age in the year of the flood, we get 187 + 182 + 600 = 969. Did Noah not bring his grandfather on the ark? *** Answer?

Part II Mathematicians

In this second part we are going to look at the mathematical contributions of those who have made a commitment to religious life.

Euclid in Book 7 of the *Elements*, defines a **perfect number** as that which is equal to its own parts. For example, 6 = 1 + 2 + 3.

Geometrically this is [__1__2___3____] [____6____]

The divisors of 6 are 1, 2, and 3. Do not include 6. 1+2+3 = 6

Question to ponder: can you think of another number, less than 50 that is perfect?

28 1+2+4+7+14

In Book 9, Proposition 36, Euclid proves:

If as many numbers as we please beginning from an unit be set out continuously in **double** proportion until the **sum of all become prime**, and if the **sum multiplied into the last number** makes some number, then the product will be perfect.

To clarify:

- 1 + 2 = 3 prime $\rightarrow 2 \times 3 = 6$ perfect
- 1 + 2 + 4 = 7 prime $\rightarrow 4 \times 7 = 28$ perfect
- 1 + 2 + 4 + 8 = 15 not prime
- 1 + 2 + 4 + 8 + 16 = 31 prime $\rightarrow 16 \times 31 = 496$ perfect
- 1 + 2 + 4 + 8 + 16 + 32 = 63 not a prime
- 1 + 2 + 4 + 8 + 16 + 32 + 64 = 127 prime \rightarrow 64 x 127 = 8128 perfect

6, 28, 496, and 8128 were the only perfect numbers that Euclid would have known, which makes the Proposition 36 even more remarkable.

Hrotsvit of Gandersheim 935-1000 in Saxony

A Benedictine Nun

One of the earliest European Playrights



Sapientia, a play, has the following lines appear:

I should not leave unmentioned the principal numbers Those which are called "**perfect numbers**." These have parts which are neither larger nor smaller than the number itself, such as the number six, whose parts, three, two, and one, add up to exactly the same sum as the number itself. For the same reason **twenty-eight**, **four hundred ninety-six**, and **eight thousand one hundred went-eight** are called perfect numbers.

These are the same perfect numbers known to Euclid.

- In a 1997 article in the *Mathematics Teacher*, entitled, *Even Perfect Numbers: (Update)2*, Stanley Bezuszka and Margaret Kenney update an earlier article about perfect numbers.
- This article provides a segue in this part of our talk. Father Bezuszka is a Jesuit priest, on the faculty of Boston College, who has been extremely active in the field of mathematics and mathematics education. In the article we are introduced to many people who were involved with perfect numbers.
- *** Needs more information or transition

- Euclid's Book 9, Proposition 36 in modern notation:
 - If $1 + 2 + ... + 2^{n-1} = 2^n 1$ is a prime, then $2^{n-1}(2^n - 1)$ is a

Fact: n must be a prime for the sum to be prime

Challenge: Not all $2^p - 1$ are prime if p is prime.

- Prime numbers in the form $2^p 1$ are called Mersenne primes, after the French monk, Marin Mersenne.
- Perhaps his most significant contribution to the history of mathematics was the fact that he served as a conduit for the spread of mathematical ideas.
- Corresponded with Fermat, Descartes, Pascal, Huygens, Galileo, Robertval, and many others, thus acting as a stimulus to scientific and mathematical research.



Marin Mersenne (1588-1648)

Mersenne worked on problems involving perfect numbers with mixed results.

We know from Euclid that if $2^p - 1$ is a prime then $2^{p-1}(2^p - 1)$ is a perfect number.

Thus every Mersenne prime leads to a perfect number.

Research on perfect numbers today is directed at finding Mersenne primes.

Alcuin of York 735 - 804

> Cathedral School of York Student Teacher Headmaster Retired to become the abbot of St. Martin of Tours Monastery



- In 781 he was invited by Charlemagne to direct his palace school. His work at the school led to major reforms in education and set the stage for the revival of learning in Europe.
- In 796 he retired from the palace school and became the abbot of the St. Martin of Tours Monastery. He died in 804.
- Alcuin himself wrote numerous textbooks in arithmetic, geometry and astronomy. *Propositiones ad acuendos juvenes (Problems to sharpen the young).* This text contains probably the earliest collection of problems written in Latin.

De emptore in C dinariis A merchant and 100 pence

A merchant wanted to buy a hundred pigs for a hundred pence. For a boar he would pay 10 pence; and for a sow 5 pence; while he would pay one penny for a couple of piglets. How many boars, sow, and piglets must there have been for him to have paid exactly 100 pence for the 100 animals?

De emptore in C dinariis A merchant and 100 pence

Solution: Take 9 sows and a boar, to the value of 55 pence altogether, and 80 piglets at 40 pence, which bring the total to 90. For the other 5 pence take 10 piglets, and that brings the total of pigs and pence to 100.

De homine qui obviavit scolaris A man meeting some scholars

A man met some scholars and said to them: " How many are there in your school?" One of them answered: " I don't want to tell you that. Count us twice and multiply by three. Then divide into four parts. If you add me to the fourth part, you will have a hundred." How many did he meet walking on the road?

De homine qui obviavit scolaris

A man meeting some scholars

Solution: Twice thirty-three is 66. This is the number that he met walking . Twice this number is 132. This multiplied by three makes 396. The fourth part of this is 99. Adding the boy who answered makes 100.

De duobus hominibus boves ducentibus Two men leading oxen

Two men were leading oxen along a road and one said to the other; "Give me two oxen and I'll have as many as you have." Then the other said: "Now you give me two oxen and I'll have double the number you have." How many oxen were there, and how many did each have?

De duobus hominibus boves ducentibus Two men leading oxen

Solution: The one who asked for two oxen to be given to him had 4, and the one who was asked had 8. The latter gave two oxen to the one who requested them, and each had 6. The one who first received now gave back two oxen to the other who had 6 and so now had 8 which is twice 4, and the other was left with 4 which is half 8.

De sode et scrofa A breeding sow and a pigsty.

A farmer created a new yard in which he put a breeding sow, which produced a litter of 7 piglets in the centre, which with the mother makes 8. They bear litters of 7 piglets each in the first corner of the yard; then all of them bear litters of 7 each in the next corner and so on for all four corners. Finally they all bear litters of 7 in the centre pigsty. How many pigs are there now altogether, including the mothers?

De sode et scrofa A breeding sow and a pigsty.

Solution: At the first birth, in the centre pigsty, there are 7 piglets and the mother makes eight. Eight eights are 64. That is the number of pigs, including mothers in the first corner. The eight sixty-fours are 512. That is the number of pigs in the second corner. Again eight times 512 is 4096. That is the number of pigs and mothers in the third corner. If one multiplies this by eight there are 32,768. That is the number of pigs in the fourth corner. Multiplying this by eight gives 262,144. That is the total number of pigs after the final litters are produced in the centre pigsty.

De obitu cuiusdam patrisfamilias A dying man's will

A dying man left 960 shillings and a pregnant wife. He directed that if a boy was born he should receive 9/12 of the estate and the mother should receive 3/12. If however a daughter was born, she should receive 7/12 of the estate and the mother should receive 5/12. It happened however that twins were born, a boy and a girl. How much should the mother receive, how much the son, how much the daughter?

De obitu cuiusdam patrisfamilias A dying man's will

Solution: 9 and 3 makes 12 and 12 ounces make a pound. Similarly 7 and 5 also make 12: and twice 12 is 24. 24 ounces make two pounds, that is 40 shillings. Hence divide 960 shillings into 24 parts; the twenty-fourth part is 40. Then take nine parts of 40 shillings. Now the mother takes three parts compared to the son and five compared to the daughter, and 3 and 5 make 8. Then take 8 parts of 40. The mother receives eight 40s, that is 16 pounds, which is 320 shillings. Then take what remains, which make seven parts, seven parts of 40, which is 14 pounds or 280 shillings. This is what the daughter receives. Adding 360 and 320 and 280 gives 960 shillings or 48 pounds.

This answer is wrong!

Giovanni Saccheri (1667-1733)

Italian Jesuit Priest

Author of *Euclides ab omni Naevo Vindicates* (Euclid freed from every flaw)

Unknowingly proved many theorems of non-Euclidean geometry.

Gauss, Bolyai and Lobachevski are given credit for discovering non-Euclidean geometry a century later.

When Euclid wrote the *Elements* in 300 B.C., he set down a list of postulates for geometry. His fifth postulate attracted the most attention.

Postulate V. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles.

In textbooks today this would be stated as: Given a line and a point not on the line, there exist exactly on line through the point parallel to the given line.
This postulate is much longer and more complex than the other postulates of Euclid. It almost seems like it should be a theorem.

Euclid may have felt the same way. He did not use this postulate until Theorem 29 of Book I.

Many important theorems of Euclidean Geometry flow from this postulate.

Many geometers attempted to fix the problem of Postulate V, including Saccheri.

Saccheri started with a quadrilateral ABCD, with AD and BC being equal in length and perpendicular to AB.



- It is easy to prove, without using Postulate V that angles ADC and BCD are equal . There are then three possibilities:
- 1. The angles are obtuse Hypothesis of the obtuse angle
- 2. The angles are right
- 3. The angles are acute

Hypothesis of the obtuse angle Hypothesis of the right angle Hypothesis of the acute angle

The hypothesis of the right angle is Euclidean geometry, the only geometry. The plan was to eliminate the hypotheses of the obtuse and the acute angle.

Saccheri quickly disposed of the hypothesis of the obtuse angle. He spent over two hundred pages attacking the hypothesis of the acute angle. He proved theorem after theorem, hoping to come up with a contradiction. In the end he almost literally waved his hands and said that the hypothesis of the acute angle was wrong. He thought that he had freed Euclid from every flaw, although he was perhaps to good a logician to really believe it. What is actually contained is this book are many, many theorems of non-Euclidean geometry. It would take others with a different focus to realize this.

- Rabbi ben Ezra (1092 1167)
 - Born and raised in Spain
 - Inspiration for Robert Browning's "Rabbi Ben Ezra"
- Spread the work of Arabs through Europe He is generally given credit for introducing Europe to the Indian symbols:
 - 1 2 3 4 5 6 7 8 9

Levi ben Gershon (1288 - 1344)

- French Rabbi
- Philosopher
- Astronomer
- Scientist
- **Biblical commentator**
- Mathematician
- **Prolific Author**
 - Many about math, all in Hebrew

- Levi's Work included:
 - Earliest rigorous use of Mathematical Induction Combinatorics
 - Algorithms for square and cube root extraction Solutions to simultaneous linear equations
 - Attempted proof of Euclid's 5th postulate
 - Trigonometry
 - Harmonic Numbers
 - Inventor of Jacob's Staff

Maaseh Hoshev (The Art of Calculation), 1321

This text focused primarily on proof by induction

Without modern notation he proved: $(1 + 2 + 3 + ... + n)^2 = 1^3 + 2^3 + 3^3 + ... + n^3$

Modern Proof

 $(1 + 2 + 3 + ... + n)^2 = 1^3 + 2^3 + 3^3 + ... + n^3$ Recall 1 + 2 + 3 + ... + n = n(n+1)/2So $(1/4)n^2(n + 1)^2 = 1^3 + 2^3 + 3^3 + ... + n^3$ Let n = 1 $(1/4)(1)^2(1+1)^2 = 1^3$ 1=1 True

Assume that it is true for *k*.

 $(1/4) k^2 (k + 1)^2 = 1^3 + 2^3 + 3^3 + \dots + k^3$

Therefore

$$\begin{array}{l} (1/4) \ k^2 \ (k+1)^2 + (k+1)^3 = 1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 \\ (1/4) \ (k+1)^2 \ [k^2 + 4(k+1)] \\ (1/4) \ (k+1)^2 \ [k^2 + 4k+4)] \\ (1/4) \ (k+1)^2 \ (k+2)^2 \\ (1/4) \ (k+1)^2 \ ([k+1]+1)^2 \end{array}$$

Therefore the statement works for k + 1 integers.

Therefore the statement holds true for all integer values of *n*. $(1/4)n^2(n + 1)^2 = 1^3 + 2^3 + 3^3 + ... + n^3$

Jacob's Staff was a surveying instrument.

The pole was marked in degrees, and the altitude of the stars could be determined by using a sliding wooden panel on the rod



John Venn (1834-1923)

Raised in a family of clergy

Given credit for the use of Euler Circles



• For example:



Part III Religious Use of Mathematics

Gregorian Calendar

- Pope Gregory VIII
 - Commissioned mathematicians and astronomers and clerics to revise the Julian Calendar.
 - Revision was necessary due to the leap year
 - Issue: Tropical year (average interval between vernal equinoxes) and Julian Calendar out of sync by 1 day, every 131 years.
 - Essentially the date for Easter needed to easily be determined.

Current International Standard

- Christopher Clavius (1537-1612) receives most credit for the reform.
- In 1582 it was determined that the length of a tropical year was 356.2422

Converting the decimal .2422



$$\frac{1}{4}, \frac{7}{29}, \frac{8}{33}, \frac{31}{128}, \frac{132}{545}, etc$$

The first value suggests that we add a leap day every four years.

The second suggests that we add seven leap days every 29 years.

All discussion ended in 1972 when Coordinated Universal Time became effective internationally.

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